Jordan Alexander Key (b. 1990): “To Say Pi,” a Black-MIDI Ballet (electro-acoustic)
Premier: December 8th and 9th, 2018; Washington, District of Columbia, The Dance Place
Further Performances:

- May 17th and 18th, 2019; Kennedy Center for the Performing Arts, Washington, District of Columbia.

Audio Recording: https://www.youtube.com/watch?v=XTJNmSJ_-aU

Commissioner:	National Science Foundation in conjunction with Drexel University
Engineering Team:	Leslie Lamberson and Drexel University School of Engineering
Choreographer:	Lucy Bowen McCauley
Dance Company:	Bowen McCauley Dance Company
Music Composition:	Jordan Alexander Key
Backdrop Art:	Jordan Alexander Key

“To Say Pi” (Circular Cantor Set over Mountain-Scape)

Presentation Notes:

To Say Pi brings together multidisciplinary efforts in material science, applied mechanics, art, dance, and music. The goal of the outreach portion of this NSF grant project proposes “novel means to reach untapped local communities at all ages… enabled through a dance-mechanics education and outreach program,” highlighting “the innate creativity and correlations involved in [the sciences and fine arts], and aims to inspire the next generation of STEAM (science, technology, engineering, arts and mathematics) enthusiasts.”

This piece – in visuals, dance, and sound – attempts to embody concepts of numerical irrationality, particularly the number π (approximately 3.1415…). An irrational number is simply any number that cannot be represented precisely by a ratio of two whole numbers (a/b). With its nonterminating decimal expansion, pi cannot be completely expressed by a finite fraction. Pi is most commonly learned as an intrinsic aspect of circles and trigonometric functions, usually first encountered in elementary school with the formula for the area of a circle, $A=\pi r^2$, or the circumference length of a circle, $C=2\pi r$, where r is the length from the center of the circle to any point on its edge.

The name of the music comes from a quotation from English essayist, Daniel Tammet, in his book, *Thinking in Numbers: How Maths Illuminates Our Lives*. Tammet writes, “A bell cannot tell time, but it can be moved in just such a way as to say twelve o’clock – similarly, a man cannot calculate infinite numbers, but he can be moved in just such a way as to say pi.” Our ability to say “pi” and have an internal pointer to such an abstract concept, a concept embedded in the infinite and infinitesimal and intrinsic to the fabric of our cosmos, is quite miraculous. While the ratio of a circle’s circumference to its diameter seems rather trivial and a simple calculation, since its proposal over two thousand years ago this abstract ratio has inspired and humbled humanity in its apparent omnipresence and seeming inexpressibility and incommensurability. The remarkable thing about humanity, however, is that we have persisted over the millennia to attempt this ostensibly impossible utterance of profound truth.

Saying 3.1415… in the language of mathematics or through descriptions of geometry is clearly nothing new, easily traced back to ancient Greece, Babylon, Egypt, China, and India. However, can these utterances be translated into other, less concretely mathematical languages: motion, tone, time, etc.? While all aspects of our universe are ultimately ruled by mathematics, some realms – art, music, and dance as some examples – are less frequently associated with the quantitative sciences. The Ancient Greeks, however, recognized an intrinsic link between mathematics and music, understanding the mathematical truth made physical in the strumming of strings and the whispering of flutes. This mini-ballet attempts to find avenues for new translations of mathematical irrationality into sound, recognizing both the capability of sound to make the abstract concrete as well as the ultimate impossibility to fully “say pi.”

As such, *To Say Pi* attempts to bring together not only mathematics and music in a non-trivial way but also bridge numerous musical genres, looking both to music's past and the future of music in the 21st century. Just as composer Conlon Nancarrow realized the limitations of human performability and consequently sought new modes of expression in the mechanical performers of his day – player pianos – *To Say Pi* seeks modes of expression in the digital capabilities of the computer. Nearly half a century ago, Nancarrow attempted to build his own mechanized orchestra and ultimately failed, but now at every composer's disposal are the nearly infinite possibilities of MIDI in conjunction with synthesized and sampled sound. Unfortunately, many of MIDI's digital possibilities to supersede typical styles of performance have often been under utilized. Thus, *To Say Pi* exploits the digital orchestra allowed through MIDI not merely to synthetically replicate what human orchestras can do or create sounds wholly unique to the digital world, but rather to create a music of familiar sounds organized in a truly superhuman fashion, pointing to a new "third-stream" music in the space between sub-genres of digital pop music, like Black-MIDI, and 20th century classical music, like Sound Mass, New Complexity, and Post-Minimalism.

To sonify pi, the composer has superimposed various approximations of and recursive calculating processes for pi onto a few musical dimensions, particularly rhythm, time, and harmony. For example, a relatively good approximation of pi is 355/113. If one takes these numbers as frequency values measured in Hertz, two pitches and their implied overtone constituents can be used to harmonically suggest the "sound" of pi. Another close rational approximation with much smaller numbers is 22/7. While such values can not be easily heard as pitches if interpreted as Hertz values, the frequency of these values can be heard as "beats" and can stand for polyrhythmic structures: 22 equally timed notes in one voice in the same time as 7 equally timed notes in the other.

Many rational approximations like those above, from relatively precise to the grossly oversimplified, have been used to generate harmonic as well as rhythmic material in this piece. Some of the rational approximations you may hear include 3/1, 13/4, 16/5, 19/6, 22/7, 44/14, 333/106, 355/113. The most accurate of these - 3/1, 22/7, 333/106, 355/113 - come from the "continued fraction" approximation of pi. At the end of the piece, you will even hear the next two approximations in the "continued fraction" algorithm: 103993/33102 and 104348/33215. Of all of these, 355/113 is the only fraction in the sequence that gives more exact digits of pi (i.e. 7) than the number of digits needed to approximate it (i.e. 6). Consequently, this ratio is the most important in the formal structure of the music.

As a consequence of using these relatively incommensurate ratios, you might occasionally feel that the piece's harmonies sound slightly "out of tune." It, in fact, is perfectly in-tune, however the sounds have been tuned to a different system of tuning. Rather than our standard Equal Temperament, the composer frequently uses temperaments representative of either precise frequency ratios (355/113 and multiples of it) or one representative of a Pythagorean Tuning system focusing on the overtone ratios of those proportions listed above. Furthermore, by using these incommensurate ratios to generate rhythmic strata, you might occasionally feel that the different layers of the music do not sound temporally “together;” however, they are precisely together, albeit in a relatively complex manner.

In the course of the piece, you might notice a few recurring melodic motives, which eventually are iterated and concatenated at different pitch levels to create melodies. The first distinctive melodic idea is initially heard in a cello at about 2 minutes into the piece. This motive was simply created by mapping equal-tempered pitches onto the graph of the Moderreva-Leibniz series of alternating reciprocals of odd numbers. This musical idea is only a fragmentary motive, since this sequence converges rather rapidly to a close approximation of pi. However, this motive is transformed and concatenated with itself at different pitch levels to create broader melodic ideas later in the piece.

Throughout the piece, we undulate between oversimplified approximations and more precise approximations, giving the music a sense of acceleration and deceleration, which attempts to simulate the elliptical motion, such as that of the planets around the sun. Ellipses and circles became the formal design of the work both concretely and abstractly. During the piece, you might notice four large sections, each with its own instrumental colors: first, metallic sounds forming a digital gamelan; second, plucked sounds forming a digital orchestra; third, more abstract tones and waves making a digital organ; fourth, all these ensembles combined. The development of the materials in each section (lasting approximately 2.5 minutes each) follows a figurative circular trajectory: building from simple structures, adding more rhythmic layers, morphing harmonies from equal temperament to “pi-temperament,” and then returning to simplicity with the diffusing of these materials. Within each of these large formal circles are smaller formal circles which follow similar trajectories as the larger ones. The temporal length of both these macro and micro formal circles follows the ratios given above: the largest formal circles designed to encompass a polyrhythmic structure of 355/113 or 333/106 and the smaller ones designed to encompass 22/7 or 44/14.

Ultimately, each of these rational ratios is an approximation. How far we take this approximation is generally determined by how precise one needs to be. In school, one commonly uses the decimal approximation 3.14 or the rational approximation 22/7. When building precision dependent computers and software and the machines reliant upon them (i.e. satellites, telescopes, quantum computers, etc.), the approximations must be even more precise. The choice is somewhat arbitrary as to how far one will take the endless process of irrational computation. The conclusion of this piece attempts to point to this possibly infinite process and the precision which it demands.

After the final circle, the coda of the piece transforms all the polyrhythmic, pi-approximating ratios, currently present in each of the three ensembles, from the realm of beat and pulse into the realm of pitch and harmony by processing the data to transform the rhythmic ratios 3/1, 13/4, 16/5, 19/6, 22/7, 44/14, etc. into 333/106, 355/113, 103993/33102 and 104348/33215. This process creates a different effect than simply generating a pitch at these frequencies. Rather, this process takes the material (all pitches, rhythms, and harmonies) and essentially accelerates it to such an extent that the music becomes only perceptible as pitch with overtones. Essentially, the “musical data,” relatively unpacked for the listener in the first 10 minutes of the piece, becomes repacked into about 30 seconds of sound at the end. This sound mass stands as another approximations of pi, but also attempts to show the computational and auditory challenge of fully realizing this number sonically and then processing it by the human ear. In this last 30 seconds, you hear as many notes as you did in the prior 10 minutes, amounting to more than one million polyrhythmic, micro-tuned musical data points.

To Say Pi was commissioned under the National Science Foundation’s grant-1751989, awarded to Dr. Leslie Lamberson and sponsored by Drexel University. Special thanks to Lucy Bowen McCauley, choreographer of the dance component of this mini ballet.

Short Program Notes:

To Say Pi focuses on a seemingly simple aspect of mathematics that makes its way into every part of our lives. We often first hear of pi as $\frac{circumference\ of\ circle}{diameter\ of\ circle}$ or only casually referenced in formulas like that of the area of a circle: $A = \pi r^2$. Pi lives in all things circular, cylindrical, and spherical in nature as a result, but maybe less clear is that it is also apparent in periodic things. For instance, the orbits of the planets around the sun and the sun around the galactic core are governed by mechanics that rely on pi.

One of the most surprising aspects of pi that this piece focuses on is its irrationality. There exists no pair of integers a and b such that $\frac{a}{b} = \pi$. We can approximate it as well as we like, but this was not even proven to be true until the 1760s by Johann Lambert. For reference, modern calculus was largely finished by 1687. There were hints of the number’s own intractability as far in the past as the ancient Egyptians. They determined values in terms of continued fractions and noted that they seemed to be able to improve on these values without much thought. In the 14th and 15th centuries, the Indian mathematician, Madhava determined a certain infinite sum would converge to pi. 200 years later, Liebniz would make the same discovery in Europe. Both the continued fraction representation and the Madhava-Liebniz Series, along with some less important representations make their way into the fabric of this piece. These mathematical structures inform and create melodies and rhythm to the limit of the perception of the human ear. Utilizing these two particular mathematical structures also ties together the work of many cultures: African, Indian, and European across many generations and emphasizes the depth of history and collaboration by numerous, often nameless, people that gave us our current understanding of pi.

One recurrent thought in both popular culture and my mind as I wrote this piece was “How can math communicate in a way that is beyond words and perhaps universally intelligible?”. One of my sincere hopes is that if an alien intelligence were to watch this piece carefully enough they would walk away understanding that humans know something deep and beautiful about the relationship of pi to the cosmos.